Name: Student number

Computational Science 260
Midterm Exam

Nov. 23

Marks

1. Let P be "grass is green", Q be "ice is hot", and R be "the year has 400 **10**
days". Express the statement $(P \wedge \neg Q) \Rightarrow (\neg P \vee R)$ in English. Also, find
the truth value of this expression.

2. A *group* is defined as a set, together with an operation which is associative, **15**
has an identy element, and an inverse for each element. For the following
three cases, state whether or not the set is a group. In particular, state if
the operation in question is associative, and indicate the identity element.
Also, indicate how the inverse is normally expressed.

   1. **N** with the operation $+$.

   2. **Z** with the operation $+$.

   3. **Z** with multiplication.

3. Is it possible that the union of two symmetric relation, both of which are **8**
defined on the same set A, is not symmetric? If your awswer is negative,
give an explantion in English. if your answer is positive, give an example.

4. Give a formal proof for $\exists x \forall y P(x, y) \vdash \forall y \exists x P(x, y)$. **15**

5. Let $A = \{1, 2, 4\}$, and let $B = \{3, 4, 6\}$. Find $(A - B) \times (B - A)$. **10**

6. Write Prolog predicates for the following definitions. Note that Prolog **16**
accepts the normal relational operators, such as $>$, $>=$, and so on.

   1. Write a predicate `wealthy(Age,Income)` which succeds if the income
     exceeds 1000 times the age.

   2. Write a predicate `tax(Sale, Tax, Total)` which calculates a 14%
     tax on sales above \$1, and adds to Sales in order to find Total. If
     Sale is below \$1, the tax is zero.

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7. Use induction to show that $3n \leq n^2$ for all $n \geq 3$. Establish the base of 14 induction, and the inductive step.

8. Let R and S be two predicates. Given $r(x,y)$ is stored as a fact in the 12 data base if xRy, and that $s(x,y)$ similarly represents the relation S. Write a Prolog predicate rcircs(X,Y) which succeeds if (X,Y) are in the relation $R \circ S$.

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———————— The End ————————

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